



## Macroscopic Traffic Flow Characterization for Stimuli Based on Driver Reaction

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### Abstract

The design and management of infrastructure is a significant challenge for traffic engineers and planners. Accurate traffic characterization is necessary for effective infrastructure utilization. Thus, models are required that can characterize a variety of conditions and can be employed for homogeneous, heterogeneous, equilibrium and non-equilibrium traffic. The Lighthill-Whitham-Richards (LWR) model is widely used because of its simplicity. This model characterizes traffic behavior with small changes over a long idealized road and so is inadequate for typical traffic conditions. The extended LWR model considers driver types based on velocity to characterize traffic behavior in non lane discipline traffic but it ignores the stimuli for changes in velocity. In this paper, an improved model is presented which is based on driver reaction to forward traffic stimuli. This reaction occurs over the forward distance headway during which traffic aligns to the current conditions. The performance of the proposed, LWR and extended LWR models is evaluated using the First Order Upwind Scheme (FOUS). The numerical stability of this scheme is guaranteed by employing the Courant, Friedrich and Lewy (CFL) condition. Results are presented which show that the proposed model can characterize both small and large changes in traffic more realistically.

**Keywords:** Lighthill Whitham and Richards (LWR) Model; Driver Reaction; Traffic Stimuli; First Order Upwind Scheme.

## 1. Introduction

Traffic models are employed to predict vehicle behavior and are essential to the design of effective control strategies [1]. Changes in traffic velocity occur due to driver interactions and can result in significant spatial changes in density. Traffic flow is impacted by leading as well as adjacent vehicles [2] and driver interactions are due to forward and lateral changes in traffic. The minimum distance between consecutive vehicles to avoid an accident is called the forward distance headway. Large changes in velocity occur with a small forward headway, and vice versa. The distance a vehicle maintains with adjacent vehicles is called the lateral distance headway. A small lateral distance headway tends to reduce traffic flow and vice versa [3]. Thus, traffic models should consider both the lateral and forward distance headways to adequately characterize traffic flow.

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Traffic flow can be categorized according to road conditions and is described as homogeneous or heterogeneous, equilibrium or non-equilibrium. In homogeneous traffic, parameters such as velocity and headway do not vary spatially [5] and vehicles follow lane discipline. Heterogeneous traffic consists of motorized and non-motorized vehicles and lane discipline is often not followed [6]. In an equilibrium flow, velocity is a function of density and the distance between vehicles is inversely proportional to density. Thus, the distance between vehicles is small when the density is large and vice versa. In a non-equilibrium flow, the velocity and distance between vehicles are not based on density [7].

Macroscopic traffic models consider the cumulative behavior of traffic including density, velocity and flow and are widely employed due to their low computational complexity and simplicity [4]. Greenshields proposed a linear relationship between traffic velocity and density [8] which can be expressed as:

$$v(\rho) = v_m \left(1 - \frac{\rho}{\rho_m}\right) \quad (1)$$

Where  $v_m$  is the maximum velocity,  $\rho$  is the density and  $\rho_m$  is the maximum density. Greenberg considered a logarithmic relationship between velocity and density [9]. However, the free flow velocity of this model tends to infinity at low densities which is not realistic. Underwood employed an exponential relationship between the velocity and density [10]. The limitation of this model is that the velocity approaches zero for large densities. The Greenshields model has been modified to provide a more realistic characterization of traffic flow [11, 12], and this is commonly employed in traffic flow modeling.

Lighthill and Whitham and Richards proposed a first-order macroscopic traffic flow model which is called the The Lighthill-Whitham-Richards (LWR) model [13, 14]. It is based on vehicle conservation on a highway which can be characterized by temporal changes in traffic density and spatial changes in flow. This model is given by:

$$\frac{\partial \rho}{\partial t} + \frac{(\partial \rho v(\rho))}{\partial x} = 0 \quad (2)$$

Where  $v(\rho)$  is the equilibrium velocity distribution [31]. The LWR model is the simple to implement [15] and is adequate for small changes in traffic flow [16, 17]. However, it has limitations in characterizing non-equilibrium traffic such as when there are large changes in flow [18]. Whitham improved the LWR model [16] by incorporating the time headway which is required during velocity alignment. Coscia [19] improved the LWR model by incorporating the delay in driver reaction to a change in traffic conditions. Daganzo [20] considered the effect of lanes and classes of vehicles on flow. However, this model assumes instantaneous traffic alignment which is unrealistic. Chanut and Buisson [21] also incorporated classes of vehicles, particularly cars and trucks, but it is difficult to characterize the classes. Drivers presumption to changes ahead is called anticipation [22], and relaxation is the alignment to changes in traffic flow. The movement of vehicles towards equilibrium conditions is called diffusion [23]. Ismael [24] proposed a macroscopic model based on forward distance headway. This model includes anticipation and diffusion terms but cannot characterize traffic behavior during congestion.

Most macroscopic models are based on only homogeneous traffic [25]. However, the LWR model was modified in [26] to consider large traffic velocity changes at bottlenecks. Another variation considers driver response as quick or sluggish in non lane discipline traffic [45]. The response of quick drivers is reduced by leading slow vehicles. To avoid speed reduction, quick drivers overtake slow vehicles when there is sufficient space between two leading vehicles. This model can be expressed as:

$$\frac{\partial \rho_z}{\partial t} + \sum_{p=1}^A c_{zp} \frac{\partial \rho_p}{\partial x} = 0 \quad (3)$$

Where  $z$  represents the driver type,  $A$  is the number of types,  $\rho_z$  is the density of vehicles having type  $z$  drivers,  $\rho_p$  is the density of vehicles having type  $p$  drivers, and  $c_{zp}$  is the response of type  $z$  drivers to the presence of type  $p$  drivers ahead which is given by:

$$c_{zp} = v_z \sigma_{zp} + \rho_p \frac{\partial v_z}{\partial \rho_p} \quad (4)$$

Where  $\sigma_{zp}$  is 1 if  $z = p$  and 0 if  $z \neq p$ , and  $v_z$  is the speed density relationship for driver type  $z$ :

$$v_z = v_{fz} \left( - \left( \frac{\rho_z}{\rho_0} / 2 \right)^2 \right) \quad (5)$$

Where  $v_{fz}$  is the free flow speed of type  $z$  drivers and  $\rho_0 = 50$  veh/km. While this model can characterize non lane discipline traffic, it ignores traffic stimuli. Thus, the density behavior of this model with different driver types is

similar to that of the LWR model. Further, the effect of vehicle interactions is not considered and an aggregate response is used for different speeds which is not an accurate characterization of traffic flow.

Traffic aligns to spatial changes in density based on the forward and lateral distances headways between vehicles. In congestion, these headways are small. Driver response is the reaction to changes in forward traffic stimuli. A driver is more responsive and alignment is quick with small headways. Conversely, a driver is less responsive and alignment is slow with large headways. A driver maintains a safe forward distance headway  $h$  and lateral distance headway  $b_s$  during alignment. The ratio of lateral distance headway to safe lateral distance headway is a stimulus for driver reaction. This reaction is quick for a large traffic stimulus and vice versa. The LWR model characterizes traffic similarly for different conditions which causes unrealistic behavior. In this paper, a macroscopic model is proposed to characterize traffic conditions based on driver reaction and traffic stimuli. Traffic alignment is assumed to occur over the distance headway. The proposed, LWR and extended LWR models are evaluated over a straight road using the First Order Upwind Scheme (FOUS) to illustrate the advantages of our approach.

The remainder of this paper is organized as follows. The proposed model is presented in Section II. The numerical solution and stability analysis are given in Section III. The performance of the proposed, LWR and extended LWR models is evaluated in Section IV and some conclusions are drawn in Section V.

## 2. Traffic Flow Modeling

Traffic can be characterized considering driver behavior and traffic flow theory [43, 44]. The steps in developing a traffic flow model are shown in Figure 1. First, a framework is built based on qualitative statements and behavioral assumptions [27]. Second, the related literature and physical laws are used to propose a traffic model [28]. Then the performance of the proposed model is evaluated numerically [29]. This is typically achieved by discretizing the model. These results are used to modify the model if necessary to improve the traffic characterization. The remainder of this section presents the traffic model.

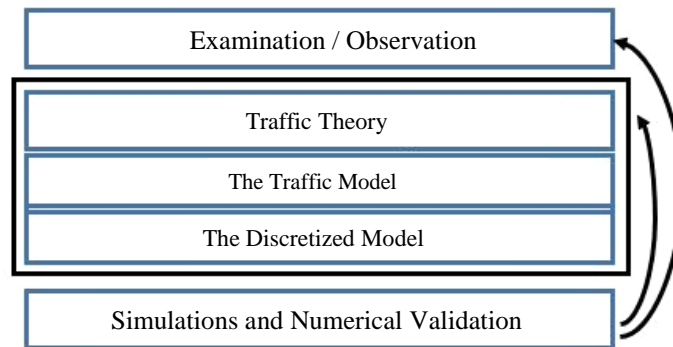


Figure 1. The steps in the development of a traffic flow model

The LWR model given in Equation 2 is based on the conservation of vehicles [30]. It is known that this model cannot characterize large changes in flow [18], and these changes can be significant when there are many driver interactions. Velocity alignment is based on forward changes in density [32] and driver reaction occurs during the time headway  $\tau$ . Further, velocity aligns according to the equilibrium velocity distribution [4]. The corresponding acceleration can be expressed as:

$$a = \frac{\Delta v}{\tau} \quad (6)$$

Where  $\Delta v$  is the change in velocity during alignment. The distance headway is covered during time headway and aligns to the equilibrium velocity of the forward traffic. The rate of change in alignment during  $\tau$  is the transition velocity given by:

$$v_t = \frac{d}{\tau} \quad (7)$$

Substituting  $\tau$  from Equation 7 in Equation 6 gives:

$$a = \frac{v_t \Delta v}{d} \quad (8)$$

The driver reaction to the change in forward conditions is:

$$v_r = at \quad (9)$$

Where  $t$  is the interaction time. Vehicle interaction is large during congestion so a large change in  $v_r$  is expected. During free flow, interactions are low and  $v_r$  should be small. From Herman and Prigogine (1979) [33] we have:

$$t = \frac{1}{v_t} \quad (10)$$

For a small  $t$  the transition velocity is large and alignment is quick. Thus, more changes in traffic and larger variations in flow are expected. For a large  $t$ , the transition velocity is small and alignment is slow. Then there are fewer changes so the flow is smooth. Substituting Equations 8 and 10 in Equation 9 gives:

$$v_r = \frac{\Delta v}{d} \quad (11)$$

Traffic aligns to the equilibrium velocity  $v(\rho)$  according to the forward conditions so that:

$$\Delta v = v(\rho) - v$$

and this gives:

$$v_r = \frac{v(\rho) - v}{d} \quad (12)$$

The forward distance headway between vehicles can be expressed as:

$$d = h + v_t \tau \quad (13)$$

Where  $h$  is the safe distance to perceive forward traffic changes [34]. Substituting this in Equation 12 gives:

$$v_r = \frac{v(\rho) - v}{h + v_t \tau} \quad (14)$$

This indicates that a driver reacts and aligns to forward conditions while covering the forward distance headway.

Lateral distance headway is the safe distance between two adjacent parallel vehicles. This can be used to characterize traffic heterogeneity [35]. As the lane width decreases, this headway decreases. Further, the lateral distance is small in the presence of large vehicles, in congestion, and when lane discipline is not followed. In free flow conditions, the lateral distances between vehicles is large. With heterogeneous traffic, the changes in flow are small and tend to zero for small lateral distances which makes vehicles vulnerable to accidents. The stimulus  $M$  for driver reaction is ratio of the actual lateral distance  $b_a$  to the safe lateral distance  $b_s$ :

$$M = \frac{b_a}{b_s} \quad (15)$$

When  $M$  is less than 1, traffic maneuverability is restricted so only small changes in traffic occur. This is called an inactive bottleneck and results in slow spatial and temporal traffic evolution. For a typical traffic flow:

$$b_a = b_s,$$

so that  $M = 1$ , while for free flow traffic:

$$b_a > b_s,$$

so that  $M > 1$ . Driver response is the product of reaction Equation 14 and stimulus Equation 15 which gives [7]:

$$\frac{v(\rho) - v}{h + v_t \tau} M \quad (16)$$

This indicates that whenever there is a change in stimulus, velocity alignment occurs according to the leading conditions over the distance headway  $h$ . The proposed model is then obtained by substituting Equation 16 as  $q(\rho)$  in Equation 2 which gives:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v_r M)}{\partial x} = 0 \quad (17)$$

This characterizes traffic flow based on driver response. When the behavior is typical ( $M = 1$ ), and there are no changes in traffic conditions, so the proposed model becomes:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} \left( \frac{\rho v(\rho)}{h + v_t \tau} \right) = 0 \quad (18)$$

Which indicates a homogeneous flow. A driver maintains a safe distance headway with the leading vehicles. Further, if the forward distance headway,  $h_t + v_t \tau$  is equal to 1, then Equation 18 gives the LWR model. Thus in the LWR model, the headway is 1 regardless of the traffic conditions, which is unrealistic. Conversely, the proposed model can be employed to characterize both homogeneous and heterogeneous flows realistically.

### 3. Discretization of the Proposed, LWR and Extended LWR Models

Macroscopic traffic flow models are partial differential systems. Typically there is no analytic solution due to abrupt changes in traffic flow [36], so they are solved using numerical approximations [37]. The first order upwind scheme (FOUS) [38] is used here to obtain numerical solutions for the proposed and LWR models. This technique is less complex than the central difference, downwind, Lax-Friedrichs and Leap-Frog schemes [39]. In this scheme, a uniform computational grid is obtained by dividing the solution space spatially and temporally  $(x, t)$ . The width of a road segment is  $\Delta x$  which is the difference between two consecutive points in the  $x$  direction while  $\Delta t$  is the time step in  $t$  direction. The density is approximated over equidistant road segments  $(x_i + \Delta x, x_i - \Delta x)$  and then approximated over the time interval  $(t_{n+1}, t_n)$ , where  $t_{n+1} - t_n = \Delta t$ .

The traffic models are approximated by spatial derivatives of flow and temporal derivatives of density, respectively. The forward in time density approximation is:

$$\frac{\partial \rho(t_n, x_i)}{\partial t} = \frac{\rho_i^{n+1} - \rho_i^n}{\Delta t} \quad (19)$$

and backward in space flow approximation is:

$$\frac{\partial q(t_n, x_i)}{\partial x} = \frac{q_i^n - q_{i-1}^n}{\Delta x} \quad (20)$$

Substituting Equations 19 and 20 in Equation 2 gives:

$$\rho_i^{n+1} = \rho_i^n - \frac{\Delta t}{\Delta x} (q(\rho_i^n) - q(\rho_{i-1}^n)) \quad (21)$$

Where the flux  $q(\rho_i^n)$  for the LWR model at the  $i$ -th time step and  $n$ -th road segment is:

$$\rho_i^n v(\rho_i^n) \quad (22)$$

and the flux  $q(\rho_{i-1}^n)$  at the  $(i - 1)$ -th time step and  $n$ -th road segment is:

$$\rho_{i-1}^n v(\rho_{i-1}^n) \quad (23)$$

Then the density at the  $i$ -th time step and  $(n + 1)$ -th road segment with the LWR model is approximated as:

$$\rho_i^{n+1} = \rho_i^n - \frac{\Delta t}{\Delta x} (\rho_i^n v(\rho_i^n) - \rho_{i-1}^n v(\rho_{i-1}^n)) \quad (24)$$

and the density at the  $i$ -th time step and  $(n + 1)$ -th road segment with the extended LWR model is approximated as:

$$\rho_{z_i}^{n+1} = \rho_{z_i}^n - \sum_{p=1}^A c_{zp} \frac{\Delta t}{\Delta x} (\rho_{p_i}^n) - (\rho_{p_{i-1}}^n) \quad (25)$$

The flux  $q(\rho_i^n)$  for the proposed model at the  $i$ -th time step and  $n$ -th road segment is:

$$\rho_i^n \frac{v(\rho_i^n) - v_i^n}{h + v_t \tau} M \quad (26)$$

and the flux  $q(\rho_{i-1}^n)$  at the  $(i - 1)$ -th time step and  $n$ -th road segment is:

$$\rho_{i-1}^n \frac{v(\rho_{i-1}^n) - v_i^n}{h + v_t \tau} M \quad (27)$$

At the  $i$ -th time step and  $(n + 1)$ -th road segment, the density with the proposed model is approximated as:

$$\rho_i^{n+1} = \rho_i^n - \frac{\Delta t}{\Delta x} \left( \rho_i^n \frac{v(\rho_i^n) - v_i^n}{h + v_t \tau} M - \rho_{i-1}^n \frac{v(\rho_{i-1}^n) - v_i^n}{h + v_t \tau} M \right) \quad (28)$$

A numerical technique approximating a convergent dynamic system should be stable [40]. This can be guaranteed

by choosing a suitable time step  $\Delta t$ . The traffic covers a maximum distance of  $v_m \Delta t$  during a time step. To approximate the density during this time, the distance covered by the traffic should be less than a road segment so that:

$$v_m \Delta t \leq \Delta x \quad (29)$$

and rearranging gives:

$$\Delta t \leq \frac{\Delta x}{v_m} \quad (30)$$

This provides a guide to the maximum allowable time step  $\Delta t$ . In this case, the slope of the gradient is always less than or equal to 1, which is known as the Courant number given by:

$$c = v_m \frac{\Delta t}{\Delta x} \quad (31)$$

The Courant, Friedrich and Lewy (CFL) condition  $c \leq 1$  guarantees numerical stability [40]. The maximum velocity for the models is set to  $v_m = 17$  m/s, and the time and roads steps are  $\Delta t = 0.1$  s and  $\Delta x = 5$  m, respectively. The CFL condition gives:

$$c = 17 \times \frac{0.1}{5} = 0.34 \leq 1 \quad (32)$$

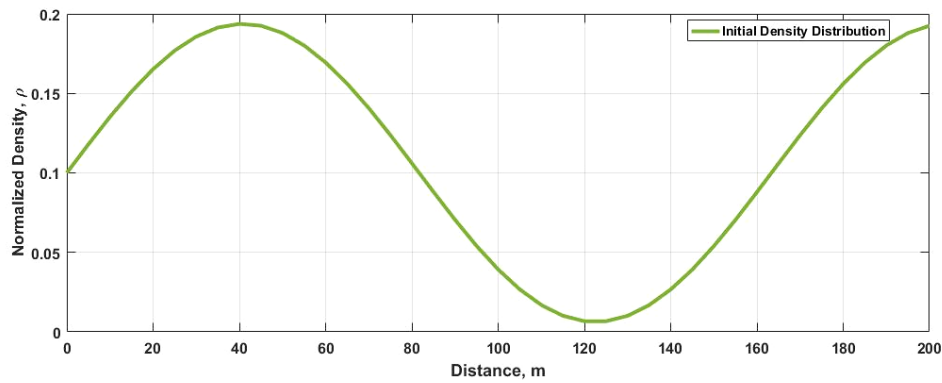
so the numerical solutions will be stable.

#### 4. Performance Results

The performance of the proposed and LWR models is evaluated over a 200 m straight road section and the extended LWR model over a 500 m section. The simulation parameters are given in Table 1. The total simulation time is 30 s, and the time and road steps are 0.1 s and 5 m, respectively. The maximum normalized density is 1 which represents a road that is 100% occupied. The maximum velocity is 17 m/s and the target equilibrium velocity distribution is Equation 1 [8] for the proposed and the LWR models, while for the extended LWR model the target velocity distribution is Equation 5. Three driver types are considered so  $A = 3$ , and the maximum speeds are 16.55, 25 and 33.33 m/s for  $z = 1, 2$  and 3, respectively [45]. The relaxation time is  $\tau = 2.5$  s and  $d = 5$  m [41-43]. The traffic stimuli [44] values are  $M = 0.5, 1$  and 1.5, and non periodic boundary conditions are employed. The initial density distribution is given in Figure 2. The density is 0.10 at 1 m, increases to 0.19 at 40 m, decreases to 0.011 at 125 m, and then increases to 0.19 at 200 m.

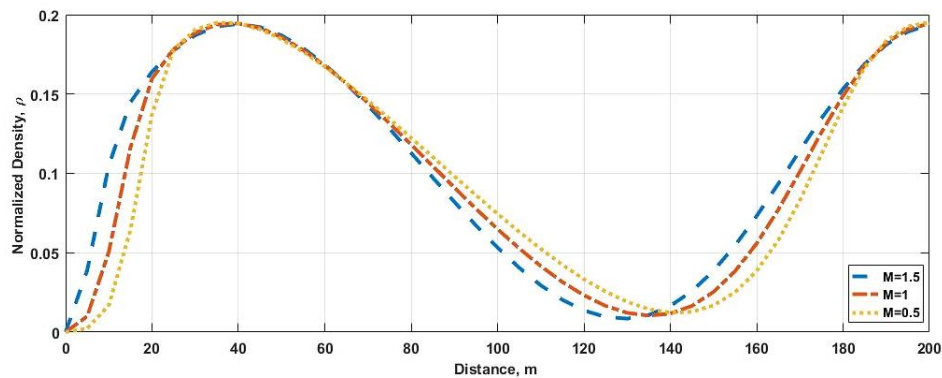
**Table 1. Simulation Parameters**

Description	Value
Simulation time	30 s
Road length for proposed and LWR models	200 m
Road length for the extended LWR model	500 m
Maximum velocity	$v_m = 17$ m/s
Maximum velocity for type 1 drivers	$v_{m1} = 16.55$ m/s
Maximum velocity for type 2 drivers	$v_{m2} = 25$ m/s
Maximum velocity for type 3 drivers	$v_{m3} = 33.33$ m/s
Time step	0.1 s
Road step	5 m
Maximum density	$\rho_m = 1$
Relaxation time	$\tau = 2.5$ s
Traffic stimulus	$M = 0.5, 1$ and 1.5



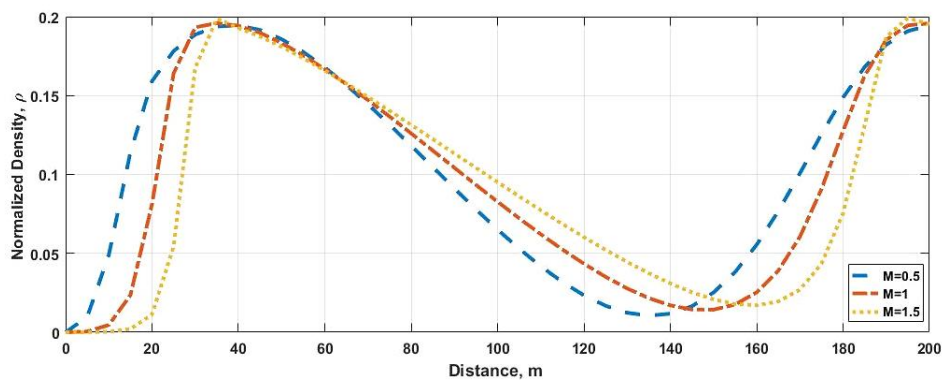
**Figure 2. Initial density distribution over a 200 m road section**

The density behavior of the proposed model at 5 s over a 200 m road section with traffic stimuli  $M = 0.5, 1$  and  $1.5$ , is given in Figure 3. With  $M = 0.5$ , the density is 0.011 at 5 m, increases to 0.19 at 40 m, decreases to 0.021 at 140 m, and is 0.19 at 200 m. With  $M = 1$ , the density is 0.021 at 5 m, increases to 0.19 at 40 m, decreases to 0.011 at 135 m, and is 0.19 at 200 m. With  $M = 1.5$ , the density is approximately 0 at 5 m, increases to 0.19 at 35 m, then decreases at 130 m to 0.021, and is 0.19 at 200 m. These results show that the density evolution varies with the lateral distance headway. With a smaller traffic stimulus this evolution is slower, whereas it is quicker with a larger stimulus.



**Figure 3. Traffic density behavior of the proposed model over a 200 m road section at 5 s with  $M = 0.5, 1$  and  $1.5$**

The density behavior of the proposed model over a 200 m road section at 10 s with  $M = 0.5, 1$  and  $1.5$  is given in Figure 4. With  $M = 0.5$ , the density is 0.011 at 5 m, increases to 0.19 at 40 m, decreases to 0.011 at 140 m, and then increases to 0.19 at 200 m. With  $M = 1$ , the density at 5 m is 0, increases to 0.19 at 40 m, and then decreases to 0.021 at 140 m. After 140 m, it gradually increases and reaches 0.19 at 200 m. With  $M = 1.5$ , the density is 0.01 at 20 m, increases to 0.20 at 35 m, and then decreases to 0.023 at 160 m. At 195 m and 200 m, it is 0.20 and 0.19, respectively. Thus, the density evolution is faster for a larger traffic stimuli and this is very evident at 10 s.

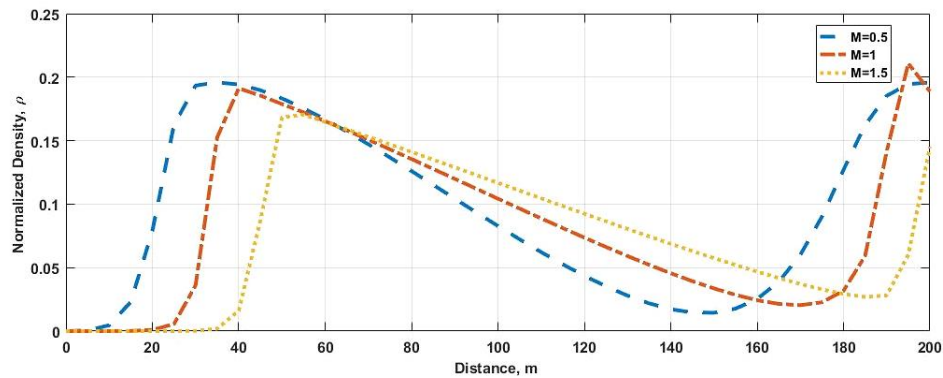


**Figure 4. Traffic density behavior of the proposed model over a 200 m road section at 10 s with  $M = 0.5, 1$  and  $1.5$**

The density behavior of the proposed model at 20 s with  $M = 0.5, 1$  and  $1.5$  is given in Figure 5. With  $M = 0.5$ , the density is 0.0041 at 10 m, increases to 0.19 at 30 m, and is 0.012 at 150 m and 0.19 at 200 m. With  $M = 1$ , at 25 m the density is 0.0011, increases to 0.19 at 40 m and then decreases to 0.022 at 175 m. It is 0.21 and 0.18 at 195 m and 200 m, respectively. With  $M = 1.5$ , the density is 0.0021 at 49 m, increases to 0.17 at 55 m, and at 190 m and 200 m, respectively.

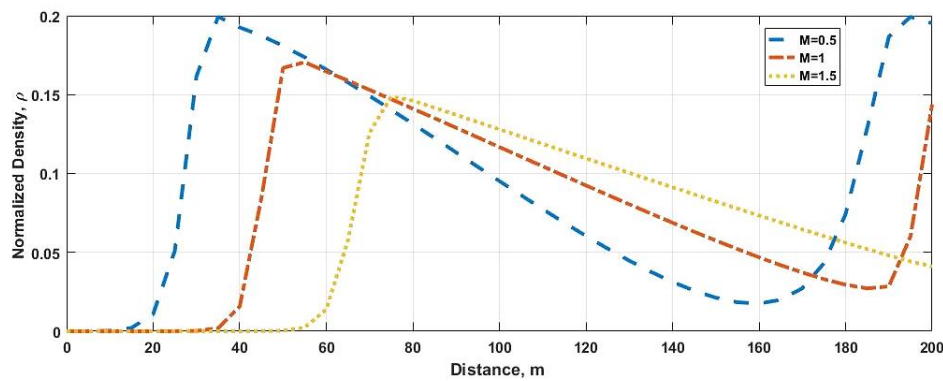


200 m it is 0.031 and 0.14, respectively. These results show that a small traffic stimulus impedes density evolution, so it is slowest with  $M = 0.5$ .



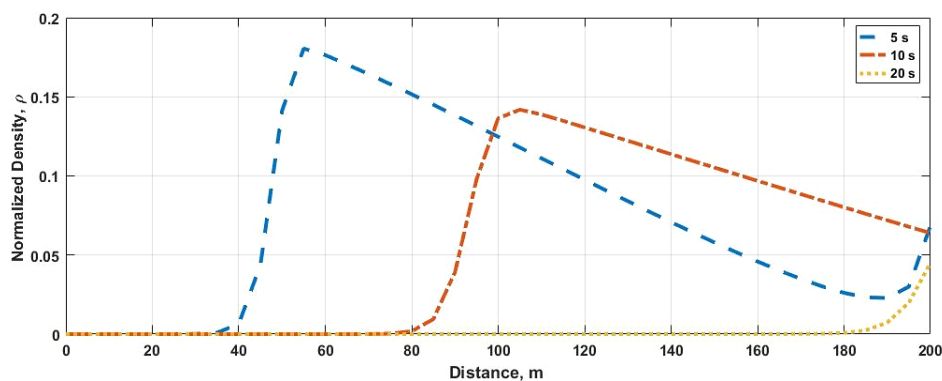
**Figure 5. Traffic density behavior of the proposed model over a 200 m road section at 20 s with  $M = 0.5, 1$  and  $1.5$**

The density behavior of the proposed model with  $M = 0.5, 1$  and  $1.5$  at 30 s is given in Figure 6. With  $M = 0.5$ , the density is 0.011 at 20 m, increases to 0.20 at 35 m, gradually decreases to 0.014 at 160 m, and then increases to 0.19 at 200 m. With  $M = 1$ , the density is 0.0021 at 40 m, increases to 0.17 at 55 m, gradually decreases to 0.034 at 190 m, and then increases to 0.14 at 200 m. With  $M = 1.5$ , the density is 0.011 at 60 m and gradually decreases to 0.041 at 200 m. These results show that the difference in density with a small and large traffic stimulus increases over time.



**Figure 6. Traffic density behavior of the proposed model over a 200 m road section at 30 s with  $M = 0.5, 1$  and  $1.5$**

The density behavior of the LWR model over a 200 m road section at 5 s, 10 s and 20 s is given in Figure 7. At 5 s, the density is 0.050 at 42 m, increases to 0.18 at 55 m, then decreases to 0.024 at 190 m, and is 0.068 at 200 m. At 10 s, the density is 0.010 at 80 m and increases to 0.13 and 0.14 at 100 m and 105 m, respectively. It is 0.061 at 200 m. At 20 s, the density is 0.0021 and 0.042 at 185 m and 200 m, respectively. Thus, the density behavior with the LWR model is similar for all traffic stimuli, which is unrealistic.



**Figure 7. Traffic density behavior of the LWR model over a 200 m road section at 5, 10 and 20 s**

The density behavior of the proposed model for 30 s over a 200 m road section with  $M = 0.5, 1$  and  $1.5$  is shown in Figures 8 to 10, respectively. These figures show that the temporal and spatial evolution of traffic density is slow with  $M = 0.5$ , faster with  $M = 1$  and quick with  $M = 1.5$ . The corresponding density behavior of the LWR model is



given in Figure 11. Comparing this with Figures 8 to 10, the LWR model temporal and spatial density evolution is quicker than the proposed model even with  $M = 1.5$ . Further, the proposed model characterizes traffic more realistically as the density changes according to the traffic stimulus, whereas the LWR model provides the same results for all stimuli.

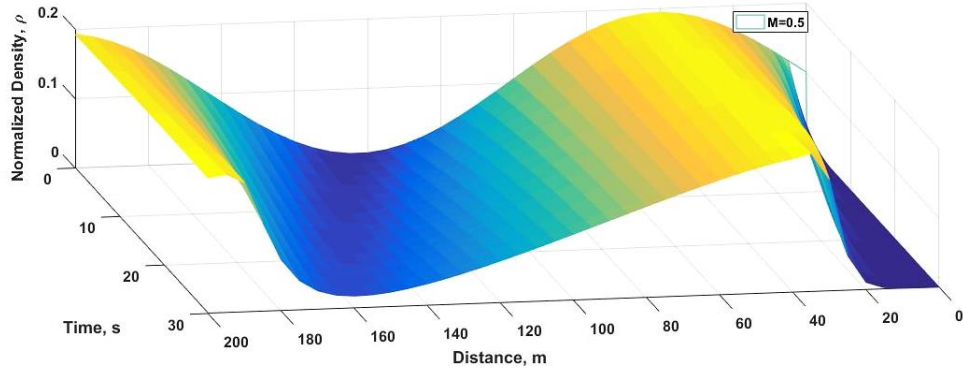


Figure 8. Traffic density behavior of the proposed model with  $M = 0.5$  for 30 s

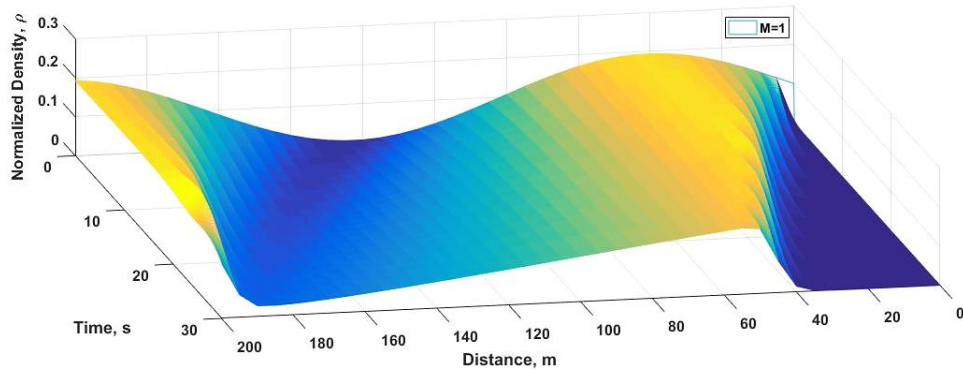


Figure 9. Traffic density behavior of the proposed model with  $M = 1$  for 30 s

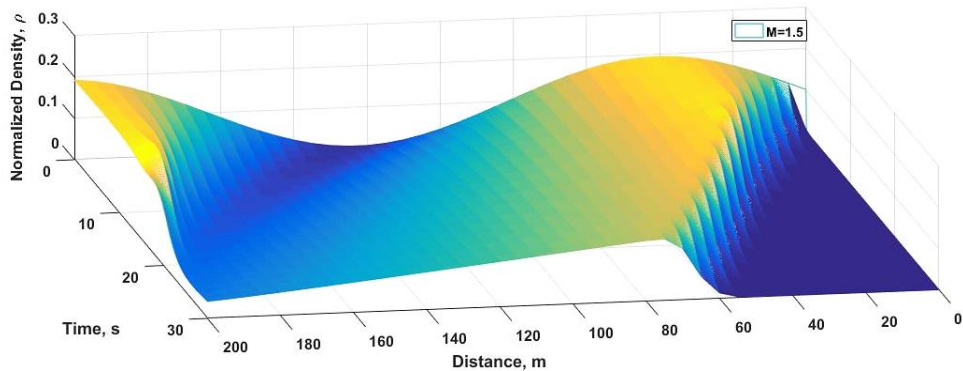


Figure 10. Traffic density behavior of the proposed model with  $M = 1.5$  for 30 s

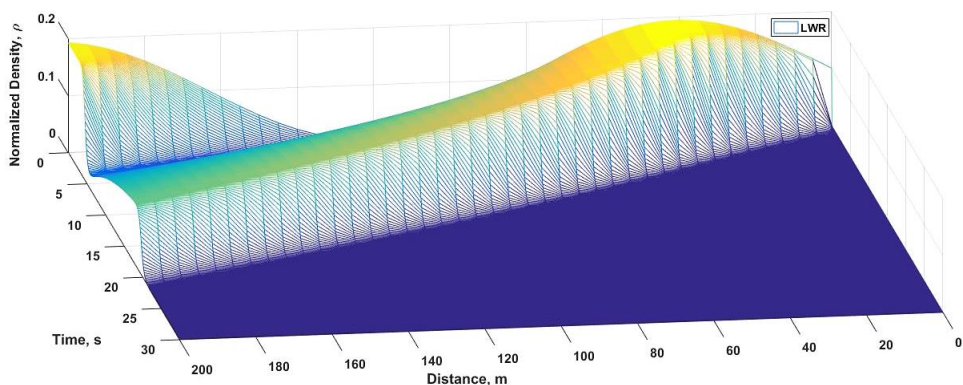
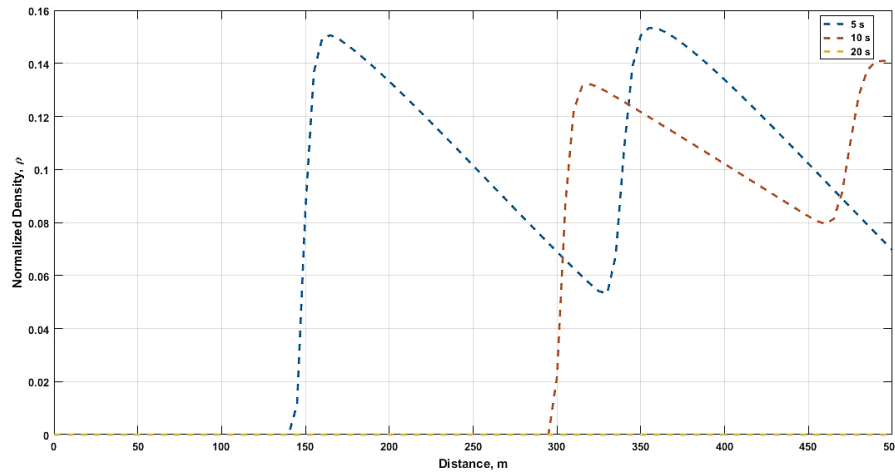


Figure 11. Traffic density behavior of the LWR model for 30 s over a 200 m road section

The density behavior of the extended LWR model over a 500 m road section with three user types and  $v_{fz} = 16.55, 25$  and  $33.33$  m/s at 5, 10 and 20 s is shown in Figure 12. At 5 s, the density is 0.011 at 140 m and increases to 0.15 at 165 m. It is 0.05 at 340 m, 0.15 at 355 m and 0.070 at 500 m. At 10 s, the density increases from 0 at 295 m to 0.13 at 320 m. It is 0.080 at 460 m and 0.14 at 500 m. At 20 s, the traffic has left the 500 m road section. Unlike the proposed and LWR model behavior shown in Figures 6 and 8 respectively, traffic moves very fast with the extended LWR model. Further, there are large changes in density with the extended LWR model at 5 s between 150 m and 340 m and between 340 m and 500 m.



**Figure 12.** Density behavior of the extended LWR model with three driver types and  $v_{fz} = 16.55, 25$  and  $33.33$  m/s, on a 500 m straight road section at 5, 10 and 20 s

The traffic density behavior over a straight road section was evaluated for the proposed, LWR and extended LWR models. The results for the proposed model show that the density evolution is smooth and varies with the traffic stimulus. With  $M = 0.5$ , this evolution is slow due to the small lateral distances between the vehicles, and is faster with  $M = 1$  and  $M = 1.5$  as shown in Figures 4 to 6. Thus, this model can characterize traffic behavior with different driver reactions. Conversely, the LWR model does not consider driver reaction as Figure 7 shows that the behavior is the same for different conditions. Further, Figure 12 shows that the extended LWR model exaggerates traffic behavior as the vehicles move very fast regardless of the driver type.

## 5. Conclusion

The LWR model is widely employed to model traffic behavior, but it cannot accurately characterize traffic flow for large changes in traffic. This is because it does not have the flexibility to model the effects of different traffic stimuli, which is unrealistic. The extended LWR model was developed to overcome some of the shortcomings of LWR model by incorporating the effects of different driver types in non lane discipline traffic. However, this model does not consider interactions between vehicles. A new macroscopic model was proposed which characterizes the traffic flow based on driver reaction. This is motivated by the fact that drivers react when a traffic stimuli is noticed. Results were presented which show that the traffic evolution with the proposed model varies with the traffic stimuli. The temporal and spatial evolution is slow with  $M = 0.5$ , faster with  $M = 1$  and quick with  $M = 1.5$ . Thus, the proposed model improves on the LWR and extended LWR models and can be employed to characterize both small and large changes in traffic flow.

The design and management of infrastructure remains a significant challenge for traffic engineers and planners. An advantage of the proposed model is that it can characterize various traffic conditions. It can be employed for homogeneous, heterogeneous, equilibrium and non-equilibrium traffic. Further, it can be employed in autonomous vehicles to aid in traffic alignment during transitions. This model can be extended by including the lateral and forward distance headway distributions to better predict traffic flow.

## 6. Declarations

### 6.1. Data Availability Statement

The data presented in this study are available in article.

### 6.2. Funding

This Project was supported by Higher Education Commission of Pakistan under the establishment of the National Center in Big Data and Cloud Computing at the University of Engineering and Technology, Peshawar.

### 6.3. Conflicts of Interest

The authors declare no conflict of interest.

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